Propagation of waves perpendicular to the magnetic field in a two-component warm plasma

1972 J. Phys. A: Gen. Phys. 5 471

(http://iopscience.iop.org/0022-3689/5/3/017)

J. Phys. A: Gen. Phys., Vol. 5, March 1972. Printed in Great Britain

Propagation of waves perpendicular to the magnetic field in a two-component warm plasma

S R SHARMA and T N BHATNAGAR
Department of Physics, University of Rajasthan, Jaipur, India

MS received 27 July 1970, in revised form 14 June 1971

Abstract. Propagation of small amplitude waves perpendicular to the magnetic field in a two component magnetoactive, unbounded warm plasma having arbitrary mass ratio has been examined using Maxwell's equations for the electromagnetic field and the first three moment equations for each component. The full pressure tensor equation (neglecting the heat flow tensor) has been used and the effect of momentum and pressure relaxation mechanisms has been included in the analysis. Dispersion relations for pure transverse waves are obtained and discussed in detail for both low and high frequency propagation. The effect of thermal motions and collisions is examined for transverse and coupled waves. It has been found that the pressure relaxation mechanism contributes significantly in the damping of oscillations at the harmonic cyclotron frequency.

1. Introduction

In a previous paper (Bhatnagar and Sharma 1970) we have investigated the propagation of waves in a two-component warm plasma along the external magnetic field, using the first three moment equations. In the present work we extend the analysis to waves propagating perpendicular to the magnetic field taken along the z direction. The dispersion relations for pure transverse waves and coupled longitudinal–transverse oscillations propagating perpendicular to the magnetic field are derived and discussed in detail. Particular attention has been paid to the effect of thermal motions and collisions at the upper hybrid and the cyclotron harmonic frequency. The collisional damping of waves propagating at extremely low frequencies is also investigated.

2. Linearization and derivation of dispersion relations

After linearizing the basic equations (Bhatnagar and Sharma 1970) about the initial static state in the usual manner, we assume that the perturbed quantities vary as

$$f = f_0 \exp \mathrm{i}(Kx-\omega t) \tag{2.1}$$

where K is the propagation vector taken along the x direction and ω is the propagation frequency. The perturbed number density, electric and magnetic fields are, respectively, given by

$$n^{\mathrm{e,i}} = \frac{Ku_1^{\mathrm{e,i}}}{\omega}n_0 \tag{2.2}$$

$$E_j = \mathrm{i}\frac{4\pi n_0 e c^2}{\omega(\omega^2 - K^2c^2)}\left(K^2(u_1^{\mathrm{i}} - u_1^{\mathrm{e}}) - \frac{\omega^2}{c^2}(u_j^{\mathrm{i}} - u_j^{\mathrm{e}})\right) \tag{2.3}$$

$$b_j = \mathrm{i}\frac{4\pi n_0 e c}{K^2c^2 - \omega^2}\,\epsilon_{j1m} K(u_m^{\mathrm{i}} - u_m^{\mathrm{e}}). \tag{2.4}$$

Using the above relations, the components of the pressure tensor are given as follows:

$$p_{11}^{\mathrm{i}} = KP_0(a_1^{\mathrm{i}}u_1^{\mathrm{i}} + 2\mathrm{i}a_2^{\mathrm{i}}u_2^{\mathrm{i}} + \nu_{\mathrm{ie}}'a_3^{\mathrm{i}}u_1^{\mathrm{e}} + 2\nu_{\mathrm{ie}}'a_4^{\mathrm{i}}u_2^{\mathrm{e}}) \tag{2.5}$$

$$p_{12}^{\mathrm{i}} = p_{21}^{\mathrm{i}} = KP_0\left(-2\mathrm{i}a_2^{\mathrm{i}}u_1^{\mathrm{i}} + \frac{A_{\mathrm{e}}}{A}u_2^{\mathrm{i}} - 2\nu_{\mathrm{ie}}'a_4^{\mathrm{e}}u_1^{\mathrm{e}} + \mathrm{i}\nu_{\mathrm{ie}}'\frac{B}{A}u_2^{\mathrm{e}}\right) \tag{2.6}$$

$$p_{13}^{\mathrm{i}} = p_{31}^{\mathrm{i}} = \frac{KP_0}{2}\left\{\left(\frac{\omega_{\mathrm{e}}+\Omega_{\mathrm{e}}}{X} + \frac{\omega_{\mathrm{e}}-\Omega_{\mathrm{e}}}{Y}\right)u_3^{\mathrm{i}} + \mathrm{i}\nu_{\mathrm{ie}}'\left(\frac{1}{X}+\frac{1}{Y}\right)u_3^{\mathrm{e}}\right\} \tag{2.7}$$

$$p_{23}^{\mathrm{i}} = p_{32}^{\mathrm{i}} = \frac{KP_0}{2}\left\{\nu_{\mathrm{ie}}'\left(\frac{1}{X}-\frac{1}{Y}\right)u_3^{\mathrm{e}} - \mathrm{i}\left(\frac{\omega_{\mathrm{e}}+\Omega_{\mathrm{e}}}{X} - \frac{\omega_{\mathrm{e}}-\Omega_{\mathrm{e}}}{Y}\right)u_3^{\mathrm{i}}\right\} \tag{2.8}$$

$$p_{33}^{\mathrm{i}} = \frac{KP_0}{3WZ}(R^{\mathrm{e}}u_1^{\mathrm{i}} + \nu_{\mathrm{ie}}'Q^{\mathrm{i}}u_1^{\mathrm{e}}) \tag{2.9}$$

$$p_{22}^{\mathrm{i}} = p_{33}^{\mathrm{i}} - \frac{2}{W}(\Omega_{\mathrm{e}}\nu_{\mathrm{ie}}'p_{12}^{\mathrm{e}} + \mathrm{i}\Omega_{\mathrm{i}}\omega_{\mathrm{e}}p_{12}^{\mathrm{i}}) \tag{2.10}$$

where

$$a_1^{\mathrm{i}} = \frac{3\omega_{\mathrm{e}}}{W} + \frac{5\mathrm{i}}{3WZ}\{\nu_{\mathrm{i}}\omega_{\mathrm{e}}(\omega + \mathrm{i}\nu_{\mathrm{ei}}') - \nu_{\mathrm{e}}\nu_{\mathrm{ie}}'\nu_{\mathrm{ei}}'\}$$
$$+ \frac{8\omega_{\mathrm{e}}\Omega_{\mathrm{e}}\Omega_{\mathrm{i}}\nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}'B}{AW^2} + \frac{4}{AW^2}(A_{\mathrm{e}}\Omega_{\mathrm{i}}^2\omega_{\mathrm{e}}^2 - A_{\mathrm{i}}\Omega_{\mathrm{e}}^2\nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}')$$

$$a_2^{\mathrm{i}} = \frac{\Omega_{\mathrm{i}}A_{\mathrm{e}}\omega_{\mathrm{e}} + \Omega_{\mathrm{e}}\nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}'B}{AW}$$

$$a_3^{\mathrm{i}} = \frac{3\mathrm{i}}{W} - \frac{5}{3WZ}\{\nu_{\mathrm{i}}\omega_{\mathrm{e}} + \nu_{\mathrm{e}}(\omega + \mathrm{i}\nu_{\mathrm{ie}}')\}$$
$$+ \frac{4\mathrm{i}\Omega_{\mathrm{e}}\Omega_{\mathrm{i}}B}{AW^2}(\nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}' - \omega_{\mathrm{e}}\omega_{\mathrm{i}}) + \frac{4\mathrm{i}}{AW^2}(\Omega_{\mathrm{e}}^2A_{\mathrm{i}}\omega_{\mathrm{i}} + \Omega_{\mathrm{i}}^2A_{\mathrm{e}}\omega_{\mathrm{e}})$$

$$a_4^{\mathrm{i}} = \frac{A_{\mathrm{i}}\Omega_{\mathrm{e}} - \Omega_{\mathrm{i}}\omega_{\mathrm{e}}B}{AW} \qquad P_0 = P_0^{\mathrm{e}} = P_0^{\mathrm{i}} = nkT_0$$

$$X = (\omega_{\mathrm{e}} + \Omega_{\mathrm{e}})(\omega_{\mathrm{i}} - \Omega_{\mathrm{i}}) + \nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}' \qquad A = A_{\mathrm{e}}A_{\mathrm{i}} + \nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}'B^2$$

$$Y = (\omega_{\mathrm{e}} - \Omega_{\mathrm{e}})(\omega_{\mathrm{i}} + \Omega_{\mathrm{i}}) + \nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}' \qquad B = 1 - \frac{4\Omega_{\mathrm{e}}\Omega_{\mathrm{i}}}{W}$$

$$A_{\mathrm{e,i}} = \omega_{\mathrm{e,i}} - \frac{4\Omega_{\mathrm{e,i}}^2}{W}\omega_{\mathrm{i,e}} \qquad \Omega_{\mathrm{e,i}} = \frac{eB_0}{m^{\mathrm{e,i}}c}$$

$$W = \omega_{\mathrm{e}}\omega_{\mathrm{i}} + \nu_{\mathrm{ei}}'\nu_{\mathrm{ie}}' \qquad Z = \omega(\omega + \mathrm{i}\nu')$$

$$\omega_{\mathrm{e,i}} = \omega + \mathrm{i}\nu_{\mathrm{e,i}} + \mathrm{i}\nu_{\mathrm{ei,ie}}' \qquad \nu' = \nu_{\mathrm{ei}}' + \nu_{\mathrm{ie}}'$$

$$R^{\mathrm{e}} = 3Z\omega_{\mathrm{e}} + 5\mathrm{i}\nu_{\mathrm{i}}\omega_{\mathrm{e}}(\omega + \mathrm{i}\nu'_{\mathrm{ei}}) - 5\nu_{\mathrm{e}}\nu'_{\mathrm{ei}}\nu'_{\mathrm{ie}}$$
$$Q^{\mathrm{i}} = 3\mathrm{i}Z - 5\nu_{\mathrm{e}}(\omega + \mathrm{i}\nu'_{\mathrm{ie}}) - 5\nu_{\mathrm{i}}\omega_{\mathrm{e}}.$$

The pressure tensor components for the electron fluid can be obtained by interchanging 'i' and 'e' in subscript and superscript of the above relations, and changing Ω_{i} to $-\Omega_{\mathrm{e}}$.

Eliminating all the variables except u_j, we obtain two sets of independent equations, one of which is given by

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} u_3^{\mathrm{e}} \\ u_3^{\mathrm{i}} \end{pmatrix} = 0 \tag{2.11}$$

where

$$A_{11} = 1 - S^{\mathrm{e}} - \frac{K^2 c_{\mathrm{e}}^2}{2\omega}\left(\frac{\omega_{\mathrm{i}} - \Omega_{\mathrm{i}}}{X} + \frac{\omega_{\mathrm{i}} + \Omega_{\mathrm{i}}}{Y}\right)$$
$$A_{12} = S^{\mathrm{e}} - \frac{\mathrm{i}\nu'_{\mathrm{ei}} K^2 c_{\mathrm{e}}^2}{2\omega}\left(\frac{1}{X} + \frac{1}{Y}\right)$$
$$A_{21} = S^{\mathrm{i}} - \frac{\mathrm{i}\nu'_{\mathrm{ie}} K^2 c_{\mathrm{i}}^2}{2\omega}\left(\frac{1}{X} + \frac{1}{Y}\right)$$
$$A_{22} = 1 - S^{\mathrm{i}} - \frac{K^2 c_{\mathrm{i}}^2}{2\omega}\left(\frac{\omega_{\mathrm{e}} + \Omega_{\mathrm{e}}}{X} + \frac{\omega_{\mathrm{e}} - \Omega_{\mathrm{e}}}{Y}\right)$$

in which

$$S^{\mathrm{e,i}} = \frac{\omega_{\mathrm{pe,pi}}^2}{\omega^2 - K^2 c^2} - \frac{\mathrm{i}\nu_{\mathrm{ei,ie}}}{\omega}$$
$$c_{\mathrm{e,i}}^2 = \frac{P_0}{n_0 m^{\mathrm{e,i}}} \qquad \omega_{\mathrm{pe,pi}} = \left(\frac{4\pi n_0 e^2}{m^{\mathrm{e,i}}}\right)^{1/2}.$$

The other equation is given by

$$L_{j1}u_1^{\mathrm{e}} + L_{j2}u_2^{\mathrm{e}} + L_{j3}u_1^{\mathrm{i}} + L_{j4}u_2^{\mathrm{i}} = 0 \tag{2.12}$$

where $j = 1, 2, 3, 4$ and

$$L_{11} = 1 - \frac{\omega_{\mathrm{pe}}^{*2}}{\omega^2} - \frac{K^2 c_{\mathrm{e}}^2}{\omega} a_1^{\mathrm{e}} \qquad L_{12} = \frac{\mathrm{i}\Omega_{\mathrm{e}}}{\omega} - \frac{2K^2 c_{\mathrm{e}}^2}{\omega}\mathrm{i}a_2^{\mathrm{e}}$$
$$L_{13} = \frac{\omega_{\mathrm{pe}}^{*2}}{\omega^2} - \frac{K^2 c_{\mathrm{e}}^2}{\omega}\nu'_{\mathrm{ei}} a_3^{\mathrm{e}} \qquad L_{14} = -\frac{2K^2 c_{\mathrm{e}}^2}{\omega}\nu'_{\mathrm{ei}} a_4^{\mathrm{e}}$$
$$L_{21} = \frac{\omega_{\mathrm{pi}}^{*2}}{\omega^2} - \frac{K^2 c_{\mathrm{i}}^2}{\omega}\nu'_{\mathrm{ie}} a_3^{\mathrm{i}} \qquad L_{22} = -\frac{2K^2 c_{\mathrm{i}}^2}{\omega}\nu'_{\mathrm{ie}} a_4^{\mathrm{i}}$$
$$L_{23} = 1 - \frac{\omega_{\mathrm{pi}}^{*2}}{\omega^2} - \frac{K^2 c_{\mathrm{i}}^2}{\omega} a_1^{\mathrm{i}} \qquad L_{24} = -\frac{\mathrm{i}\Omega_{\mathrm{i}}}{\omega} - \frac{2K^2 c_{\mathrm{i}}^2}{\omega}\mathrm{i}a_2^{\mathrm{i}}$$
$$L_{31} = -\frac{\mathrm{i}\Omega_{\mathrm{e}}}{\omega} + \frac{2K^2 c_{\mathrm{e}}^2}{\omega}\mathrm{i}a_2^{\mathrm{e}} \qquad L_{32} = 1 - S^{\mathrm{e}} - \frac{K^2 c_{\mathrm{e}}^2}{\omega}\frac{A_{\mathrm{i}}}{A}$$

$$L_{33} = \frac{2K^2c_e^2\nu'_{ei}}{\omega}a_4^i \qquad L_{34} = S^e - \frac{K^2c_e^2}{\omega}\frac{i\nu'_{ei}B}{A}$$

$$L_{41} = \frac{2K^2c_i^2\nu'_{ie}}{\omega}a_4^e \qquad L_{42} = S^i - \frac{K^2c_i^2}{\omega}\frac{i\nu'_{ie}B}{A}$$

$$L_{43} = \frac{i\Omega_i}{\omega} + \frac{2K^2c_i^2}{\omega}ia_2^i \qquad L_{44} = 1 - S^i - \frac{K^2c_i^2}{\omega}\frac{A_e}{A}$$

with

$$\omega^{*2}_{pe,pi} = \omega^2_{pe,pi} - i\omega\nu_{ei,ie}.$$

Equation (2.11) describes the propagation of a pure transverse wave in which the perturbed motion is along the direction of the external magnetic field. Equation (2.12) governs coupled longitudinal–transverse oscillations which are confined to the plane perpendicular to the magnetic field. It will be noted that the coupling is affected not only by the external magnetic field but also by the thermal motions. This additional coupling between the two types of oscillations due to thermal motions vanishes in the case when the selfrelaxation frequencies tend to infinity.

3. Transverse waves

The dispersion relation of the pure transverse waves propagating perpendicular to the magnetic field as obtained from equation (2.11) is given by

$$\omega^2 - K^2c^2 = \frac{\omega\omega_{pe}^2\{K^2c_i^2B' - \omega(1+m)A'\}}{i\nu_{ei}\{K^2c_i^2B' - \omega(1+m)A'\} - A'\omega^2 - K^4c_e^2c_i^2W + K^2\omega D(c_i^2\omega_e + c_e^2\omega_i)} \tag{3.1}$$

where

$$A' = XY$$

$$B' = \omega_i(\omega_e^2 - \Omega_e^2) + \omega_e(\omega_i^2 - \Omega_i^2) + \nu'_{ei}\nu'_{ie}(\omega_e + \omega_i) + i\nu'\nu'_{ei}\nu'_{ie} + i\nu'(\omega_e\omega_i - \Omega_e\Omega_i)$$

$$D = \omega_e\omega_i - \Omega_e\Omega_i - \nu'_{ei}\nu'_{ie} \qquad m = \frac{m^e}{m^i}.$$

If we put $m_i = \infty$ and $\nu_i = \nu_{ie} = \nu'_{ie} = \nu'_{ei} = 0$ in equation (3.1) we obtain the relation given by Sharma (1969). Neglecting all collisions, we get the relation obtained by Jaggi (1962) for the corresponding case of isotropic pressure.

Assuming $K^2S_e^2(1+m) \ll |\omega^2 - \Omega_e^2|, |\omega^2 - \Omega_i^2|$ and neglecting collisions and the term containing $S_e^2S_i^2$ or S_e^4 equation (3.1) yields the expression for the refractive index n

$$n^2 = \frac{K^2c^2}{\omega^2} = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\left(1 - \frac{\omega_{pe}^2S_e^2}{(\omega^2 - \Omega_e^2)(\omega^2 - \Omega_i^2)}\{2\Omega_i^2 - \omega^2(1+m^2)\}\right)^{-1} \tag{3.2}$$

where

$$S_{e,i}^2 = \frac{c_{e,i}^2}{c^2} \quad \text{and} \quad \omega_p^2 = \omega_{pe}^2 + \omega_{pi}^2.$$

It is seen that the wave propagates only when $\omega^2 > \omega_p^2$. It is also apparent from the dispersion relation that the external magnetic field affects wave propagation only through thermal motions of the electrons and ions. However, as the magnetic field becomes extremely large the contribution of thermal effects also approaches zero and the wave propagates with refractive index n given by the usual formula

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2}. \tag{3.3}$$

Equation (3.2) shows that the thermal motions tend to decrease the value of n^2 when

$$\Omega_i < \omega < \left(\frac{2}{1+m^2}\right)^{1/2} \Omega_i \text{ or where } \omega > \Omega_e.$$

For other values of propagation frequencies the value of n^2 is somewhat increased by the thermal motions.

If on the other hand the magnetic field approaches zero, the refractive index is given by

$$n^2 = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\left(1 + \frac{\omega_{pe}^2 S_e^2(1+m^2)}{\omega^2}\right)^{-1}. \tag{3.4}$$

For high frequency propagation equation (3.1) can be simplified. Retaining only first order terms in collision frequencies we have

$$\frac{K^2c^2}{\omega^2} = 1 - \frac{(1+m-2S_i^2)}{F}\frac{\omega_{pe}^2}{\omega^2} + i\left(\frac{S_i^2(3\nu+4\nu') - 2(1+m)(\nu+\nu')}{F} + \frac{(1+m-2S_i^2)}{F^2}G\right)\frac{\omega_{pe}^2}{\omega^3} \tag{3.5}$$

where

$$F = S_e^2 S_i^2 - S_e^2(1+m) + 1$$

$$G = (\nu+\nu')(F+1) + \nu_{ei}(1+m-2mS_e^2) - S_e^2(\nu_i + \nu'_{ie} + m\nu_e + m\nu'_{ei}).$$

Neglecting higher order thermal corrections relation (3.5) gives

$$\begin{aligned} n^2 &= \frac{K^2c^2}{\omega^2} \\ &= 1 - \frac{\omega_{pe}^2}{\omega^2}\{1+m+S_e^2(1+m^2)\} + i[\nu_{ei}(1+m)^2 + S_e^2\{2\nu_{ei} \\ &\quad \times (1+m+m^2+m^3) + \nu_e + \nu'_{ei}(1+m) + m^2\nu_i + \nu'_{ie}(m+m^2)\}] \times \frac{\omega_{pe}^2}{\omega^3}. \end{aligned} \tag{3.6}$$

It is apparent from equation (3.6) that the thermal motions tend to decrease the value of n^2. The contribution of the pressure relaxation mechanism in the damping of the wave is much smaller as compared to that of the momentum relaxation mechanism. It is also seen that the magnetic field does not play any role for high frequency propagation.

4. Coupled waves

The dispersion relation (2.12) can be expanded to obtain a polynomial equation in K^2 which is difficult to solve by simple algebraic methods. Assuming $\nu_e = \nu_i \to \infty$ it gives a dispersion relation for a lossy two-component plasma which maintains an isotropic pressure. For the lossless case it reduces to the dispersion relation obtained by Seshadri (1965).

Considering equation (2.12) for the case of an electron plasma

$$(m_i = \infty, \nu_i = \nu_{ie} = \nu'_{ie} = 0 \quad \text{and} \quad u_1^i = u_2^i = 0)$$

we get the following dispersion relation:

$$\left(1 - \frac{\omega_{pe}^{*2}}{\omega^2} - \frac{K^2 c_e^2}{\omega^2}\gamma\right)\left(1 + \frac{i\nu_{ei}}{\omega} - \frac{\omega_e K^2 c_e^2}{\omega(\omega_e^2 - 4\Omega_e^2)} + \frac{\omega_{pe}^2}{K^2c^2 - \omega^2}\right) = \frac{\Omega_e^2}{\omega^2}\left(1 + \frac{2K^2c_e^2}{\omega_e^2 - 4\Omega_e^2}\right)^2 \tag{4.1}$$

where

$$\gamma = 3\frac{\omega}{\omega_e} + i\frac{5\omega\nu_e}{3\omega_e(\omega + i\nu'_{ei})} + \frac{4\omega\Omega_e^2}{\omega_e(\omega_e^2 - 4\Omega_e^2)}. \tag{4.2}$$

As pointed out by Sharma (1969) γ may be interpreted as the effective value of the ratio of the two specific heats for the electron gas. Equation (4.2) differs from his expressions in the definition of ω_e in that here it includes the contribution from the collisions with the stationary ions as well.

Assuming $\omega^2 \ll K^2c^2$ and $\omega_{pe}^2 < \omega^2$ equation (4.1) simplifies to

$$\Omega_e^2(\omega_e^2 - 4\Omega_e^2 + 2K^2c_e^2)^2 = (\omega_e^2 - 4\Omega_e^2)\left(1 + \frac{i\nu_{ei}}{\omega} - \frac{K^2c_e^2\omega_e}{\omega(\omega_e^2 - 4\Omega_e^2)}\right) \times (\omega^2 - \omega_{pe}^{*2} - K^2c_e^2\gamma). \tag{4.3}$$

If thermal motions are neglected equation (4.3) admits two solutions

$$\omega_e^2 = 4\Omega_e^2 \tag{4.4}$$

and

$$\omega^2 = \omega_{pe}^{*2} + \Omega_e^2\left(1 + \frac{i\nu'_{ei}}{\omega}\right)^{-1}. \tag{4.5}$$

Equation (4.4) corresponds to damped oscillations at $\omega = 2\Omega_e$ and equation (4.5) is the relation for damped upper hybrid resonance oscillations. It may be noted that the effective frequency for selfcollisions is mainly responsible for the damping of oscillations at $\omega = 2\Omega_e$, whereas upper hybrid frequency oscillations suffer damping primarily because of the momentum relaxation mechanism.

Considering the correction (assuming $K^2c_e^2 \ll \Omega_e^2, \omega_{pe}^2$) due to thermal motions on the above modes, equation (4.4) using equation (4.3) becomes

$$\omega_e^2 = 4\Omega_e^2 + \frac{K^2c_e^2(12\Omega_e^2 - \omega_{pe}^2)}{3\Omega_e^2 - \omega_{pe}^2}\left(1 + i\frac{36\Omega_e^3(\nu_e + \nu'_{ei} - \nu_{ei})}{(3\Omega_e^2 - \omega_{pe}^2)(12\Omega_e^2 - \omega_{pe}^2)} + i\frac{\nu_{ei}\omega_{pe}^2}{2\Omega_e(3\Omega_e^2 - \omega_{pe}^2)} - i\frac{(\nu_e + \nu'_{ei})\omega_{pe}^2}{2\Omega_e(12\Omega_e^2 - \omega_{pe}^2)}\right). \tag{4.6}$$

The dispersion relation (4.5) in the presence of thermal motions becomes

$$\omega^2 = \omega_{pe}^{*2}+\Omega_e^2\left(1+\frac{i\nu_{ei}}{\omega}\right)^{-1}+3K^2c_e^2\left\{1-i\frac{4\nu_e+9\nu'_{ei}}{9\omega}-\frac{\Omega_e^2}{3\Omega_e^2-\omega_{pe}^2}\right.$$
$$\left.\times\left(3+i\frac{\Omega_e^2(3\nu_e+3\nu'_{ei}-12\nu_{ei})+\omega_{pe}^2(7\nu_e+7\nu'_{ei}-\nu_{ei})}{\omega(3\Omega_e^2-\omega_{pe}^2)}\right)\right\}. \tag{4.7}$$

It may be noted from (4.7) that selfcollisions cause damping only in the presence of thermal motions and that this damping is much smaller as compared to the contribution from the momentum relaxation mechanism. In case of a strong magnetic field $\Omega_e^2 \gg \omega_{pe}^2$ equation (4.7) reduces to

$$\omega^2 = \omega_{pe}^2+\Omega_e^2\left(1+\frac{i\nu_{ei}}{\omega}\right)^{-1}-i\omega\nu_{ei}-i\frac{K^2c_e^2}{9\omega}(7\nu_e+12\nu'_{ei}-12\nu_{ei}). \tag{4.8}$$

An analysis of equation (2.12) in the limit $\nu_e \to \infty$ shows that for low frequency propagation the medium behaves like a gas having an effective value of γ (the ratio of two specific heats) as

$$\gamma = \frac{5}{3}+i\frac{5(\nu_i+\nu'_{ie})}{3\omega_i}+\frac{3\omega}{\omega_i}+\frac{4\omega\Omega_i^2}{\omega_i(\omega_i^2-4\Omega_i^2)}. \tag{4.9}$$

Retaining only first order terms in ω/ω_{pi} we get

$$\gamma = \frac{10}{3}\left(1-i\frac{2}{5}\frac{\omega}{\nu_i+\nu'_{ie}}\frac{\{(\nu_i+\nu'_{ie})^2+\Omega_i^2\}}{\{(\nu_i+\nu'_{ie})^2+4\Omega_i^2\}}\right) \tag{4.10}$$

which goes to 10/3 as $\omega \to 0$.

Assuming $\nu_e \to \infty$ and retaining terms up to $x(=\omega/\omega_{pi})$ and $S_i^2(=c_i^2/c^2)$ we obtain

$$n^4A_4+n^2A_2+A_0 = 0 \tag{4.11}$$

where

$$A_4 = -ix\{\tfrac{10}{3}\beta S_i^2(1+m+H^2)+\beta_i S_i^2\, dH^2\}$$
$$A_2 = ix\{(1+m+H^2)(\tfrac{10}{3}\beta S_i^2+\beta(1+m)+\beta_i S_i^2\, d+\tfrac{10}{3}S_i^2\delta)$$
$$+\beta(1+m)(H^2+\tfrac{10}{3}S_i^2)\}-\{(1+m+H^2)(H^2+\tfrac{10}{3}S_i^2)\}$$
$$A_0 = -ix\{2\beta(1+m)(1+m+H^2)\}+(1+m+H^2)^2$$

in which

$$\beta = \frac{\nu_{ie}}{\omega_{pi}} \qquad H = \frac{\Omega_i}{\omega_{pi}} \qquad \beta_i = \frac{\nu_i+\nu'_{ie}}{\omega_{pi}}$$
$$\delta = \frac{2}{5}\frac{\beta_i^2+H^2}{\beta_i(\beta_i^2+4H^2)} \qquad d = \frac{1}{\beta_i^2+4H^2}.$$

In the limit $x \to 0$ we obtain the dispersion relation for undamped magnetosonic waves

$$\frac{K^2c^2}{\omega^2} = \frac{1+m+H^2}{H^2+(10S_i^2/3)} \tag{4.12}$$

As the frequency increases, the wave shows damping due to collisions, and obeys the dispersion relation

$$\frac{K^2c^2}{\omega^2} = \frac{1+m+H^2}{H^2+(10S_i^2/3)}\left\{1+i\left(\frac{\beta H^2[(1+m)^2-\{20S_i^2(1+m)/3\}]}{(1+m+H^2)\{H^2+(10S_i^2/3)\}^2} + \frac{4S_i^2H^2(\beta_i^2+H^2)}{3\beta_i(\beta_i^2+4H^2)\{H^2+(10S_i^2/3)\}^2}\right)\frac{\omega}{\omega_{pi}}+\dots\right\}. \quad (4.13)$$

The second term vanish as $\nu_i \to \infty$. The first term in parentheses gives the contribution only from the momentum relaxation mechanism and the second term arises because of the pressure relaxations. It is apparent from equation (4.13) that the damping is caused primarily because of the momentum relaxation mechanism and not by the pressure relaxation mechanism.

Acknowledgments

The authors are grateful to Professor P L Bhatnagar and Professor B L Saraf for their kind interest in the work.

References

Bhatnagar T N and Sharma S R 1970 *J. Phys.* A: *Gen. Phys.* **3** 404–12
Jaggi R K 1962 *Phys. Fluids* **5** 949–62
Seshadri S R 1965 *Radio Sci. J. Res. Natn. Bur. Std. D* **69** 579–97
Sharma S R 1969 *Can. J. Phys.* **47** 345–51